

What is Claimed:

1                   1.     A method for calculating a greatest common divisor of a first  
2     binary integer, U, and a second binary integer, V, the method comprising the steps  
3     of:

4                   a) selecting 2M most significant bits of U as a first value  $U_{2M}$  and  
5     selecting 2M corresponding bits of V as a second value  $V_{2M}$ , dividing  $U_{2M}$  by  $V_{2M}$  and  
6     storing an integer portion of the result as a value Q;

7                   b) determining a value T as U minus the quantity Q times V;

8                   c) if T is less than zero, applying a correction term to Q to obtain a  
9     corrected value Q' and assigning the new value for T as U minus the quantity Q'  
10    times V;

11                  d) assigning V to U and T to V; and

12                  e) repeating steps a) through d) until V equals zero, whereby the value  
13    remaining in U is the greatest common divisor of the first and second binary integers.

1                   2.     A method according to claim 1, wherein:

2                   step c) includes the step of selecting 2M most significant non-zero bits of  
3     T to define a value  $T_{2M}$ , wherein the step of applying the correction term is given by  
4     the equation:

$$5 \qquad Q' = Q - (\lfloor T_{2M} / V_{2M} \rfloor + 1); \text{ and}$$

6                   step c) further includes the step of calculating Q'', a further corrected  
7     value for Q, as the greatest integer less than the quantity U divided by V if the new  
8     value of T is less than zero.

1                   3.     A method according to claim 1, wherein the first binary integer,  
2     U, has a most significant non-zero bit at bit-position B1 and the second binary integer,  
3     V, has a most significant non-zero bit at bit-position B2, where B1 and B2 are integers  
4     and B1 is greater than B2, the method further including the steps of:

5                   subtracting B2 from B1 to obtain a difference value D;

6                   comparing D to a predetermined threshold value wherein steps a)  
7     through d) are performed only if D is greater than a predetermined threshold value;

8                   if D is not greater than the predetermined threshold, then, before step e)  
9     performing the steps of:

10                   determining values X and Y such that  $U_{2M}$  times X plus  $V_{2M}$  times  
11     Y is less than  $2^M$ ;

12                   assigning a new value to U as U times X plus Y times V; and

13                   switching the values of U and V.

1                   4.     A method according to claim 3, wherein the step of determining  
2     values X and Y such that  $U_{2M}$  times X plus  $V_{2M}$  times Y is less than  $2^M$ , includes the  
3     step of invoking a further GCD routine.

1                   5.     A method according to claim 4, wherein 2M equals 32 and the  
2     further GCD routine is a Euclid routine having a modified termination condition.

1                   6.     A method according to claim 4, wherein 2M equals 64 and the  
2     further GCD routine is a Lehmer routine having a modified termination condition.

1                   7.     A method according to claim 1, further including a method for  
2     calculating a value  $V^{-1}$  being the inverse of V modulo U, wherein:

3 step a) further includes the steps of assigning a value of zero to a  
4 temporary variable U2 and assigning a value of one to a temporary variable V2; and

5 step d) further includes the steps of determining a value T2 as U2 minus  
6 Q times V2, assigning the value in V2 to U2 and assigning the value T2 to V2;

7 whereby, at step e) when V equals zero, the value of U2 is  $V^{-1}$ .

1 8. A method according to claim 3, further including a method for  
2 calculating a value  $V^{-1}$  being the inverse of V modulo U, wherein:

3 step a) further includes the steps of assigning a value of zero to a  
4 temporary variable U2 and assigning a value of one to a temporary variable V2; and

5 step d) further includes the steps of determining a value T2 as U2 minus  
6 Q times V2, assigning the value in V2 to U2 and assigning the value T2 to V2;

7 the step of assigning a new value to U as U times X plus Y times V,  
8 further includes the step of determining the value T2 as X times U2 plus Y times V2;  
9 and

10 the step of switching the values of U and V further includes the step of  
11 assigning the value of V2 to U2 and assigning the value T2 to V2;

12 whereby, at step e), when V equals zero, the value of U2 is  $V^{-1}$ .

1 9. A method for defining a Finite field that includes encryption keys  
2 for an encryption algorithm, comprising the steps of:

3 a) selecting a first binary integer value, P, having a number of bits such  
4 that the Finite field defined as values ranging between zero and the first value are  
5 sufficient for the encryption algorithm to be secure;

6 b) determining if P is a prime number, comprising the steps of:

7 calculating a greatest common divisor of P, and a second binary integer,  
8 V, wherein V is a product of predetermined prime numbers, including the steps of:

9 b1) assigning P to a temporary variable U;

10 b2) selecting 2M most significant non-zero bits of U as a first  
11 value  $U_{2M}$  and selecting 2M corresponding bits of V as a second value  $V_{2M}$ ,  
12 dividing  $U_{2M}$  by  $V_{2M}$  and storing an integer portion of the result as a value Q;

13 b3) determining a value T as U minus the quantity Q times V;

14 b4) if T is less than zero, applying a correction term to Q to  
15 obtain a corrected value Q' and assigning the new value for T as U minus the  
16 quantity Q' times V;

17 b5) assigning V to U and T to V; and

18 b6) repeating steps a) through e) until V equals zero, whereby the  
19 value remaining in U is the greatest common divisor of the first and second  
20 binary integers;

21 c) if U is greater than one, selecting an other value for P and repeating  
22 steps b) through c) until U is equal to one;

23 d) when U is equal to one after step c), passing P to a probabilistic  
24 primality testing routine to determine if P is prime;

25 whereby when P is prime, the integers from 0 to P define the Finite  
26 field.

1 10. A method according to claim 9, wherein:

2 step b4) includes the step of selecting  $2M$  most significant non-zero bits  
3 of  $T$  to define a value  $T_{2M}$ , wherein the step of applying the correction term is given  
4 by the equation:

5 
$$Q' = Q - (\lfloor T_{2M} / V_{2M} \rfloor + 1); \text{ and}$$

6 step c) further includes the step of calculating  $Q''$ , a further corrected  
7 value for  $Q$ , as the greatest integer less than the quantity  $U$  divided by  $V$  if the new  
8 value of  $T$  is less than zero.

1 11. A method according to claim 10, wherein the first binary integer,  
2  $U$ , has a most significant non-zero bit at bit-position  $B1$  and the second binary integer,  
3  $V$ , has a most significant non-zero bit at bit-position  $B2$ , where  $B1$  and  $B2$  are integers  
4 and  $B1$  is greater than  $B2$ , the method further including the steps of:

5 subtracting  $B2$  from  $B1$  to obtain a difference value  $D$ ;

6 comparing  $D$  to a predetermined threshold value wherein steps a)  
7 through d) are performed only if  $D$  is greater than a predetermined threshold value;

8 if  $D$  is not greater than the predetermined threshold, then, before step e)  
9 performing the steps of:

10 determining values  $X$  and  $Y$  such that  $U_{2M}$  times  $X$  plus  $V_{2M}$  times  
11  $Y$  is less than  $2^M$ ;

12 assigning a new value to  $U$  as  $U$  times  $X$  plus  $Y$  times  $V$ ; and

13 switching the values of  $U$  and  $V$ ; and

14 after step e) if  $U$  is greater than 1, further processing  $U$  to remove  
15 spurious factors.

1                   12. A method according to claim 11, wherein the step of determining  
2 values X and Y such that  $U_{2M}$  times X plus  $V_{2M}$  times Y is less than  $2^M$ , includes the  
3 step of invoking a further GCD routine.

1                   13. A method according to claim 12, wherein 2M equals 32 and the  
2 further GCD routine is a Euclid routine having a modified termination condition.

1                   14. A method according to claim 12, wherein 2M equals 64 and the  
2 further GCD routine is a Lehmer GCD routine having a modified termination  
3 condition.

1                   15. A method for identifying an encryption value in a Finite field,  $F_P$ ,  
2 where P is a prime number, based on a private key PV and a received public key PB,  
3 comprising the steps of:

4                   determining a mathematical inverse of PB modulo P by performing the  
5 steps of:

6                   a) assigning P to a temporary variable U and assigning PB to a  
7 temporary variable V and assigning a value of zero to a temporary variable U2  
8 and assigning a value of one to a temporary variable V2;

9                   b) selecting 2M most significant bits of U as a first value  $U_{2M}$  and  
10 selecting 2M most significant bits of V as a second value  $V_{2M}$ , dividing  $U_{2M}$  by  
11  $V_{2M}$  and storing an integer portion of the result as a value Q;

12                   c) determining a value T as U minus the quantity Q times V;

13                   d) if T is less than zero, applying a correction term to Q to obtain  
14 a corrected value Q' and assigning the new value for T as U minus the quantity  
15 Q' times V;

16 e) determining a value T2 as U2 minus Q times V2, assigning the  
 17 value in V2 to U2, assigning the value T2 to U2, assigning V to U and T to V;  
 18 and

19 f) repeating steps a) through e) until V equals zero, whereby the  
 20 value remaining in U2 is the mathematical inverse of PB; and

21 dividing PV by PB modulo P by multiplying PV times the mathematical  
 22 inverse of PB, wherein the result is the encryption value.

1 16. A method according to claim 15, wherein:

2 step d) includes the step of selecting 2M most significant bits of T to  
 3 define a value  $T_M$ , wherein the step of applying the correction term is given by the  
 4 equation:

$$5 \quad Q' = Q - (\lfloor T_{2M} / V_{2M} \rfloor + 1); \text{ and}$$

6 step d) further includes the step of calculating  $Q''$ , a further corrected  
 7 value for Q, as the greatest integer less than the quantity U divided by V if the new  
 8 value of T is less than zero.

1 17. A method according to claim 15, wherein the variable U has a  
 2 most significant bit at bit-position B1 and the variable V has a most significant bit at  
 3 bit-position B2, where B1 and B2 are integers and B1 is greater than B2, the method  
 4 further including the steps of:

5 subtracting B2 from B1 to obtain a difference value D;

6 comparing D to a predetermined threshold value wherein steps a)  
 7 through d) are performed only if D is greater than a predetermined threshold value;

8 if D is not greater than the predetermined threshold, then, before step e)  
 9 performing the steps of:

10 determining values X and Y such that  $U_{2M}$  times X plus  $V_{2M}$   
11 times Y is less than  $2^M$ ;

12 assigning a new value to U as U times X plus Y times V and  
13 determining the value T2 as X times U2 plus Y times V2; and

14 switching the values of U and V and assigning the value of V2 to  
15 U2 and assigning the value T2 to U2.

1 18. A method according to claim 17, wherein the step of determining  
2 values X and Y such that  $U_{2M}$  times X plus  $V_{2M}$  times Y is less than  $2^M$ , includes the  
3 step of invoking a further GCD routine.

1 19. A method according to claim 17, wherein 2M equals 32 and the  
2 further GCD routine is a Euclid routine having a modified termination condition.

1 20. A method according to claim 17, wherein 2M equals 64 and the  
2 further GCD routine is a Lehmer routine having a modified termination condition.